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Introductory Statistics
Hypothesis Testing for a Single Mean

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Introductory Statistics
Hypothesis Testing

- What is a hypothesis test?
- What are the steps involved in a hypothesis test?
- What are the types of errors that could be made in a hypothesis test?
- What is a level of significance?

- As an example, suppose that I claim that I am excellent free throw shooter, making 80% or more of my free throw shots.

You watch me shoot 100 free throws, where I only make 30 shots.
Given that I make at least 80% of free throws, it is highly unlikely that I would make only 30%.
You do not believe my claim because if I only made 30%, this result would rarely occur given my claim.

- Given a claim.
- Gathered evidence.
- Assessed the evidence using the claim.

Introductory Statistics
Hypothesis Testing

- State the null and alternative hypotheses.
- State the Type I and Type II Errors for the hypotheses.
- State the level of significance (maximum acceptable α).
- Check assumptions.

gather data

- Compute the test statistic.
- Calculate the p-value.
- Compare the p-value with the level of significance. Make a decision regarding the null hypothesis.
- Draw a conclusion in terms of the problem.

Introductory Statistics
Hypothesis Testing Definitions

population mean
μ

- Null Hypothesis: (H_o) a statement of no effect or no change. This statement is assumed to be true unless sufficient evidence is gathered to reject this hypothesis. $H_0: \mu = \mu_0$
- Alternative Hypothesis: (H_a) the research hypothesis. This is the statement that one wishes to support as being true. This is done by gathering evidence against the null hypothesis. $H_a: \mu < \mu_0$
- Type I Error: an error that occurs if the null hypothesis is rejected when it is true. $H_a: \mu > \mu_0$ $H_a: \mu \neq \mu_0$
  - The probability of a Type I error is denoted as α
- Type II Error: an error that occurs if the null hypothesis is not rejected when it is false. fail to reject H_0 when H_0 is false
  - The probability of a Type II error is denoted as β

Introductory Statistics

Hypothesis Testing Definitions

	State of Nature (Truth)	
	H_o is True	H_o is False
Reject *Ho*	Type I Error (α = prob. Type I Error)	Correct Decision
Fail to Reject *Ho*	Correct Decision	Type II Error (β = prob. of Type II Error)

Introductory Statistics
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population mean
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Introductory Statistics
Hypothesis Testing Definitions

	State of Nature (Truth)	
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Power $= 1-\beta =$ prob. of rejecting a false H_0

Introductory Statistics
More Hypothesis Testing Definitions

- Test statistic: a quantity computed from sample data that depends on the value of the parameter begin tested

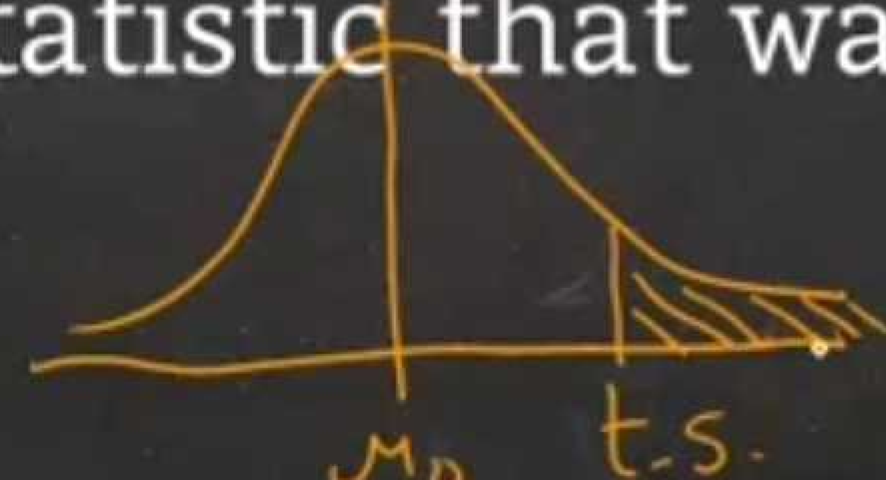
  being

  assumes H_0 is true

- Level of significance: the maximum allowable chance of making a Type I error that the researcher is willing to accept

  α

- *P*-value: the probability, computed assuming the null hypothesis is true, that a test statistic will be as or more extreme than the test statistic that was actually observed.



$H_0: \mu = \mu_0$

$H_a: \mu > \mu_0$

Introductory Statistics
Small Sample *P*-value Method: $\boxed{H_o : \mu = \mu_o}$

$$t_{obs} = \frac{\bar{y} - \mu_0}{\underbrace{s/\sqrt{n}}_{\text{estimating } \sigma \text{ with } s}}$$

$H_a : \mu < \mu_0$

P-value:

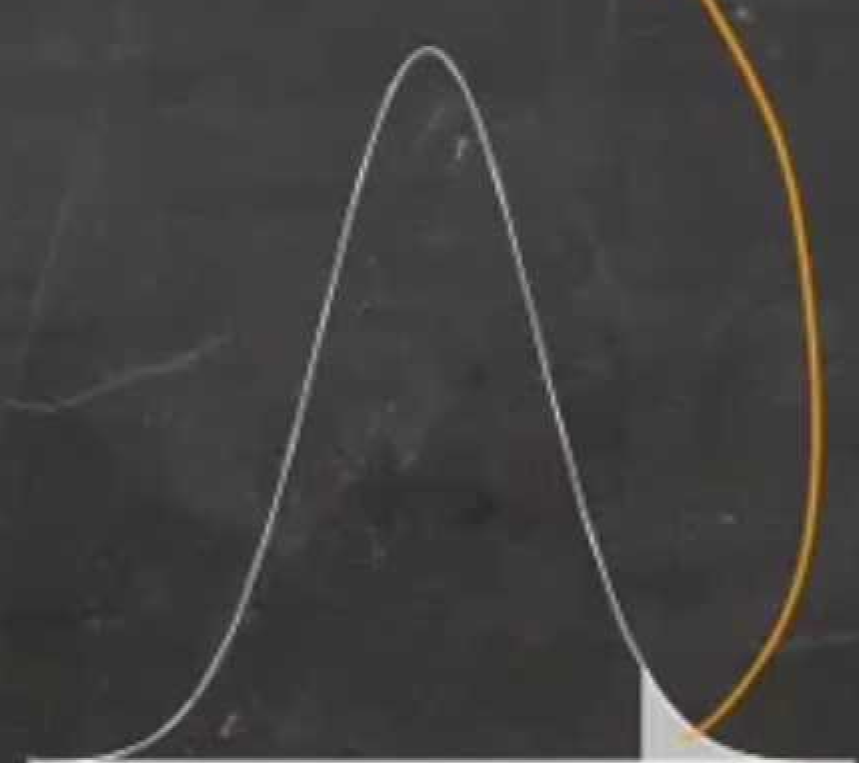
$P(T < t_{obs})$

t_{obs}

$H_a : \mu \geq \mu_0$

P-value:

$P(T \geq t_{obs})$

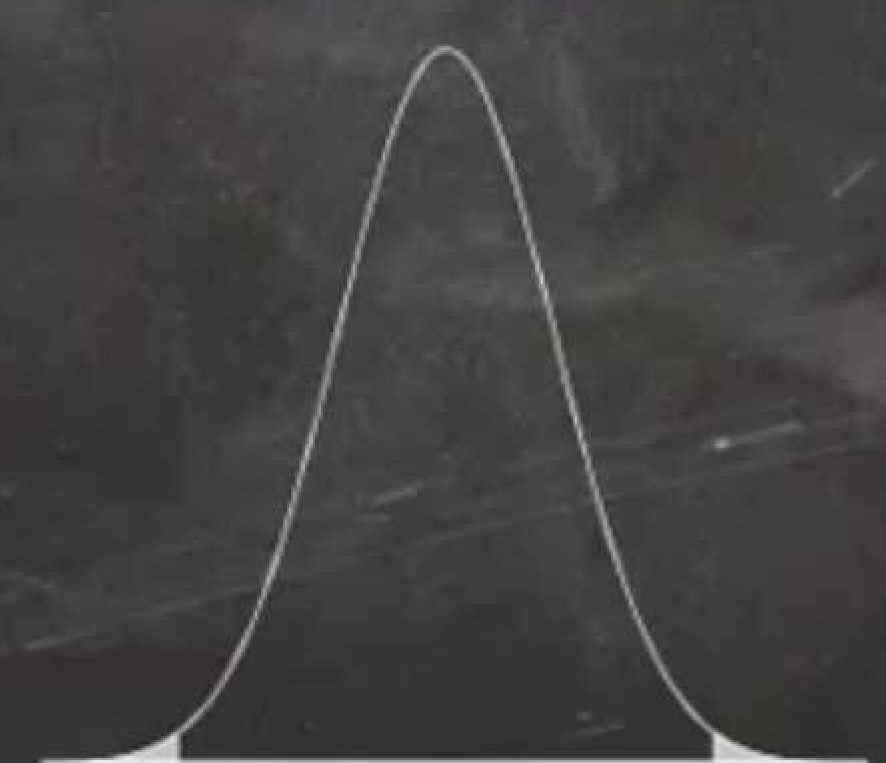


t_{obs}

$H_a : \mu \neq \mu_0$

P-value:

$2P(T \geq |t_{obs}|)$



$|t_{obs}|$

Decision Rule:

Reject H_o if $p \leq \alpha$

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P-value Method Example

Suppose that we would like to conduct a test to determine if the average Phosphorus leaching is less than 50mm. Recall that the sample mean from 32 lysimeter samples is $\bar{y}=$ 44.7166 and the sample standard deviation is $s=$ 7.8069. Use a significance level of 0.05 $=\alpha$

- State the hypotheses.

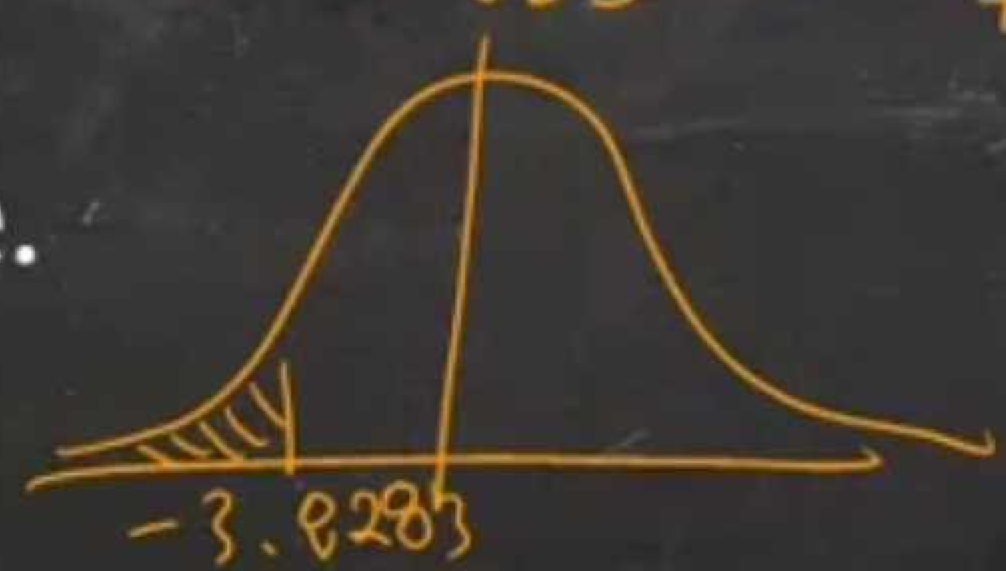
$H_0: \mu = 50\text{mm}$

$H_a: \mu < 50\text{mm}$

- Compute the test statistic.

$$t_{obs} = \frac{44.7166 - 50}{7.8069/\sqrt{32}} = -3.8283$$

- Determine the *p*-value.



$P[T_{31} < -3.8283] < 0.0005$

Introductory Statistics
P-value Method Example

Suppose that we would like to conduct a test to determine if the average Phosphorus leaching is less than 50mm. Use a significance level of 0.05.

- Make a decision regarding H_o $p \leq \alpha$ ✓

  Reject H_0

- State the conclusion in terms of the problem

  The average phosphorus leaching is significantly less than 50mm ($t(31) = -3.83$, $p < 0.0005$).

Introductory Statistics
Hypothesis Test: Phosphorus Leaching

Conduct a test to determine if the average Phosphorus leaching is less than 50mm

The TTEST Procedure

Variable: drainage

N	Mean	Std Dev	Std Err	Minimum	Maximum
32	44.7166	7.8069	1.3801	27.3436	56.7959

$\bar{y}$ s $s/\sqrt{n}$

Mean	95% CL Mean		Std Dev	95% CL Std Dev	
44.7166	-Infty	47.0566	7.8069	6.2588	10.3791

DF	t Value	Pr < t
31	-3.83	0.0003

t_{obs} P-value